Let X1, …, Xn ~ N (µ, σ^2) independent, where

* σ^2 is known and µ € R is unknown
* µ € R known and σ^2 > 0 unknown

Estimate the respective unknown parameters via the Maximum Likelihood Method.

Solution:

Let uppercase *X1, …, Xn* be i.i.d. *N (µ, σ^2)* random variables, and let lower case *xi* be the value *Xi* takes [1]. The density for each *Xi* is:

*fXi (xi)*

Because the *Xi* are independent, their joint pdf equals the sum of the separate pdf’s

*f (x1, …..., xn| µ, σ)* =

The log likelihood for the given *x1, …., xn* is:

Log *(f (x1, …..., xn| µ, σ))* = *- σ^2) - µ) ^ 2*

Since Log (*f (x1, …..., xn| µ, σ*)) is a function of the 2 variables *µ and σ* use partial derivatives with respect to *µ* and *σ^2* [2]:

* [the mean of the data] ………………………….... (1)

And

* [ *=* ]
* = [the variance of the data]

Reference:

1. <https://math.mit.edu/~dav/05.dir/class10-prep.pdf>
2. <https://bookdown.org/egarpor/inference/est-methods.html#est-methods-ml>